University of Waterloo

CO 367 Practice questions for Midterm 1

Fall 2018

Question 1 Let $A \in \mathbb{R}^{n\times n}$ be a symmetric, positive semidefinite matrix. Let λ be an eigenvalue of A and let v be a corresponding eigenvector, with $||v||_2 \leq 1$. Prove that $(A - \lambda vv^T)$ is positive semidefinite.

Question 2 Let $A, B \in \mathbb{R}^{n\times n}$ be symmetric matrices. Let $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$ be the eigenvalues of A and let $x^1, \ldots, x^n$ be corresponding eigenvectors. Let $\sigma_1 < \ldots < \sigma_n \in \mathbb{R}$ be the eigenvalues of B and let $y^1, \ldots, y^n$ be corresponding eigenvectors, with $||y^i||_2 = 1$ for all i. Let $Y \in \mathbb{R}^{n\times n}$ be a matrix whose columns are the above eigenvectors of B, i.e. $Y = [y^1|\cdots|y^n]$. Then, give an expression for the eigenvalues and eigenvectors of the matrix $(3Y^TAY)$.

Question 3 Let $f : \mathbb{R}^n \to \mathbb{R}$ be a C^2-smooth function over $\mathbb{R}^n$, and let $\alpha > 0$ be some positive constant. Prove that if $(\nabla^2 f(z) - \alpha I)$ is positive semidefinite for all $z \in \mathbb{R}^n$, then:

a) $(\nabla f(y) - \nabla f(x))^T(y - x) \geq \alpha||y - x||_2^2$ for all $x, y \in \mathbb{R}^n$.

b) f is strictly convex over $\mathbb{R}^n$.