University of Waterloo

CO 367 Practice questions for Midterm 2

Fall 2018

Question 1 Indicate, for each of the following statements, whether it is true or false. Write "True" or "False" next to each item. No explanation is necessary.

- Let $f \in C^0(\mathbb{R}^n)$. If, for all $\alpha \in \mathbb{R}$, there exists $\delta > 0$ such that $\{x \in \mathbb{R}^n : f(x) \leq \alpha\} \subseteq B_\delta(0)$, then f is coercive.
- Let $f : \mathbb{R}^n \to \mathbb{R}$ be defined by $q(x) = x^T Ax + b^T x$. If A is positive semidefinite, then q has a unique global minimizer.
- Let $f : \mathbb{R}^n \to \mathbb{R}$ be defined by $f(x) = ||Ax - b||_2^2$. If A is not positive definite, then f has no global minimizer.
- Let $f : \mathbb{R}^n \to \mathbb{R}$, and let $p \in \mathbb{R}^n$ be a descent direction at $x \in \mathbb{R}^n$. If $f(x + \alpha p) \leq f(x) + \sigma\alpha\nabla f(x)^T p$, then α satisfies the sufficient decrease condition (where $0 < \sigma < \frac{1}{2}$).
- Let f be a strictly convex quadratic function. Let Newton's method be initialized with $x^0 = 0$. Then, x^1 is the unique global minimizer for f.
- Let f be a quadratic function with a unique global minimizer $x^* \in \mathbb{R}^n$. If the trust region method is initialized with a trust region radius $\delta^0 > 0$ at a point $x^0 \in B_{\delta^0}(x^*)$, then $x^1 = x^*$.

Question 2 Consider a function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{1}{3}x^3 - 2x$.
(i) Determine all values of x^k such that x^{k+1} is well defined with Newton's method. Among those, find one that is a local minimizer.
(ii) Determine a value $r > 0$ such that if $x^0 \in B_r(x^*)$, then Newton's method converges quadratically to x^*.
(iii) Give a counter-example to the following (wrong) claim: With Newton's method, for **all** $x^k \in \mathbb{R}$, $||x^{k+1} - x^*||_2 < ||x^k - x^*||_2$.

Question 3 Let $A \in \mathbb{R}^{n\times n}$ be a symmetric matrix that is not positive semidefinite, and let $b \in \mathbb{R}^n$ be a vector that is not orthogonal to any eigenvector of A.
(i) Prove that there exists $\beta \in \mathbb{R}$ such that $\min\{x^T(A + \beta I)x + b^T x\}$ has an optimal solution $\hat{x}$ and such that $||\hat{x}||_2 = 1$.
(ii) Prove that $\hat{x}$ is a global minimizer to $\min\{x^T Ax + b^T x \;:\; ||x||_2 = 1\}$.