

CO 370 Fall 2019: Homework 2

Due: November 1st by 1:00pm

Instructions You will be graded not only on correctness, but also on clarity of exposition. You are allowed to talk with classmates about the assignment as long as (1) you acknowledge the people you collaborate with, (2) you write your solutions on your own, and (3) you are able to fully explain your solutions. In the models, always give a clear definition to your decision variables (in most cases, this means that you must explain what they represent in plain words). In case you run into trouble (a question is ambiguous, data provided have an issue, problem with the implementation, etc.), it is your responsibility to ask me or your TAs for clarifications in a timely manner.

Homework submission Questions 1, 2 and 3 are to be submitted on [Crowdmark](#). The model file for Question 4 is to be submitted on [Learn](#).

Question 1 [3 marks] Consider the following linear programming problem

$$\begin{aligned} \min & \quad \blacksquare \\ \text{s.t.} & \quad 2x_1 + x_2 + x_3 + x_4 = 5 \\ & \quad 2x_1 + 3x_2 + x_4 \leq 12 \\ & \quad x_1 - x_2 + x_3 \leq 5 \\ & \quad x_4 \geq -5 \\ & \quad x_2 - x_4 \geq 4 \\ & \quad 2x_1 + 2x_2 - x_3 - x_4 \geq 2 \\ & \quad x_1, x_2, x_3, x_4 \text{ free.} \end{aligned} \tag{P}$$

The objective function has been censored, but you know that a dual optimal solution is given by

$$y^* = \left(2, -\frac{1}{2}, 0, 5, \frac{11}{2}, 0 \right).$$

- Determine a (primal) optimal solution x^* for (P).
- Denoting the objective function of (P) by

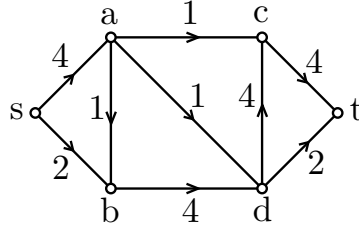
$$c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4,$$

write the dual (D) of (P).

- Determine values for c_1, c_2, c_3, c_4 such that x^* is optimal for (P) and y^* is optimal for (D).

Question 2 [4 marks]

a. Consider the following directed graph $G = (V, E)$ with capacities $u_e \geq 0$ for all $e \in E$



and the corresponding max s - t flow formulation

$$\begin{aligned}
 \max \quad & x_{sa} + x_{sb} \\
 \text{s.t.} \quad & -x_{sa} + x_{ab} + x_{ac} + x_{ad} = 0 \quad (a) \\
 & -x_{sb} - x_{ab} + x_{bd} = 0 \quad (b) \\
 & -x_{ac} - x_{dc} + x_{ct} = 0 \quad (c) \\
 & -x_{ad} - x_{bd} + x_{dc} + x_{dt} = 0 \quad (d) \\
 & x_{sa} \leq 4, \quad x_{sb} \leq 2, \quad x_{ab} \leq 1, \quad x_{ac} \leq 1, \quad x_{ad} \leq 1, \quad x_{bd} \leq 4, \quad x_{dc} \leq 4, \quad x_{ct} \leq 4, \quad x_{dt} \leq 2, \\
 & x_{sa} \geq 0, \quad x_{sb} \geq 0, \quad x_{ab} \geq 0, \quad x_{ac} \geq 0, \quad x_{ad} \geq 0, \quad x_{bd} \geq 0, \quad x_{dc} \geq 0, \quad x_{ct} \geq 0, \quad x_{dt} \geq 0.
 \end{aligned}$$

Write the dual of the above LP. All the constraints of the dual must be written individually (i.e., not in matrix notation).

b. More generally, consider any directed graph $G = (V, E)$, and any capacities $u_{qr} \geq 0$ for all $qr \in E$. The max s - t flow formulation we have seen in class is

$$\begin{aligned}
 \max \quad & \sum_{r : sr \in E} x_{sr} \\
 \text{s.t.} \quad & \sum_{r : qr \in E} x_{qr} - \sum_{r : rq \in E} x_{rq} = 0, \quad \forall q \in V \setminus \{s, t\} \\
 & 0 \leq x_{qr} \leq u_{qr}, \quad \forall qr \in E.
 \end{aligned}$$

Write the dual of this LP. All the constraints of the dual must be written explicitly (i.e., not in matrix notation).

Question 3 [4 marks] We want to decide the best possible position $x \in \mathbb{R}^2$ to place a 5G antenna. The ideal position is a known constant $b \in \mathbb{R}^2$. Unfortunately, we cannot place the antenna there, so we want to minimize the 1-norm distance

$$\|x - b\|_1 = |x_1 - b_1| + |x_2 - b_2|$$

instead. The set of possible positions is given as a list of triangles: the coordinates (x_1, x_2) of the antenna must lie in (at least) one of the triangles. The triangles are described by their vertices

$$v^{t,1}, v^{t,2}, v^{t,3} \in \mathbb{R}^2, \quad \text{for } t = 1, \dots, T,$$

where we have T triangles in total. Formulate the problem of finding the best possible position x as an integer programming problem.

Question 4 [4 marks] Implement the model of Question 3 in Julia.

You must follow the specifications given here exactly. At least one mark will be automatically subtracted if your code does not fully follow the specifications. If your code cannot be run after minor modifications, you will get 0 marks for the question.

Carefully check the following:

- Your model must be uploaded in a single Julia file (for example `hw2-model.jl`).
- Your model file must be uploaded to Learn by the project deadline.
- The data will be given to your model in the form of a data file to be included before including your model. Your model must still work if the data is changed (it will be tested with multiple variants of the data file). Therefore, the model file must not itself include("") a data file.
- Your model file is responsible for solving the model. In other words, it must contain a call to `optimize!()`.
- After the model file is run, an array named `xs` must contain an optimal solution. Note that `xs` thus cannot be the name of a model variable. Instead, you would have something similar to `xs = value.(x)` at the end of your model file, if `x` is the name of some model variable.

The following table gives the correspondence between mathematical notation and Julia names:

	Mathematical notation	Julia notation
ideal placement	b_1, b_2	<code>b[1], b[2]</code>
triangle $t = 1, \dots, T$, vertex $i = 1, 2, 3$, coordinate $j = 1, 2$	$v_j^{t,i}$	<code>v[t, i, j]</code>
optimal solution	x_1^*, x_2^*	<code>xs[1], xs[2]</code>

An example data file can be downloaded here:

<https://www.math.uwaterloo.ca/~lpoirrie/co370/d/hw2-data-00.jl>

You can plot the data and solution using this script:

<https://www.math.uwaterloo.ca/~lpoirrie/co370/d/hw2-check.jl>