# CO 370 Fall 2019: Homework 4

Due: December 3rd by 5:00pm

**Instructions** You will be graded not only on correctness, but also on clarity of exposition. You are allowed to talk with classmates about the assignment as long as (1) you acknowledge the people you collaborate with, (2) you write your solutions on your own, and (3) you are able to fully explain your solutions. In the models, always give a clear definition to your decision variables (in most cases, this means that you must explain what they represent in plain words). In case you run into trouble (a question is ambiguous, data provided have an issue, problem with the implementation, etc.), it is your responsibility to ask me or your TAs for clarifications in a timely manner.

**Homework submission** Your solutions are to be submitted on Crowdmark.

**Question 1** [15 marks] Consider the following problem:

$$\begin{array}{rrcrcrcrcrcl} \max & 2x_1 & + & 4x_2 & + & 6x_3 & + & x_4 & + & 5x_5 & & \\ \text{s.t.} & 9x_1 & + & 13x_2 & + & 7x_3 & + & 14x_4 & + & 21x_5 & \leq & 40 \\ & & & & & x \in \{0,1\}^5. & & & & & & \end{array} \tag{P}$$

Solve (P) using the branch-and-bound method. At any given node of the branch-and-bound tree, if the optimal LP solution is $\tilde{x}$ with $\tilde{x}_i \notin \mathbb{Z}$ and branching is required, always fully explore the subtree in which $\tilde{x}_i = 0$ before exploring the subtree in which $\tilde{x}_i = 1$. (In technical terms, perform a depth-first search on the branch-and-bound tree, always starting with the "$= 0$" branch.)

For every node, give the optimal LP solution and its objective function value (or write "infeasible"), and if branching is not required, specify why (pruning, infeasible, integer). Separately, draw the branch-and-bound tree.

**Question 2** [15 marks] Consider a matrix $A \in \mathbb{R}^{m \times n}$ such that

- every element $A_{ij}$ of $A$ is either 0 or 1, and
- whenever $A_{ij} = 1$ and $A_{kj} = 1$ for some $j$ and for some $i < k$, then we also have $A_{\ell j} = 1$ for all $i \leq \ell \leq k$.

Prove that $A$ is totally unimodular.