

CO 370: Deterministic Operations Research Models

Fall 2019 Outline

December 13, 2019

Objective. Operations Research (OR) is a branch of mathematics and engineering that is dedicated to solving practical decision-making problems. The process consists in two steps: First, formulate an accurate mathematical model of the problem. Then, choose an appropriate algorithm out of the optimization toolbox to provide good (if possible optimal) solutions. CO 370 covers both aspects: (1a) we will learn how to model real problems, (1b) given numerical data, we will see how to encode our models into a machine-readable form that can be fed to a solver, and (2) we will study the algorithms that are implemented in solvers. The objective is that, after taking this course, you are able to recognize, understand, model and solve OR problems on your own computer.

Prerequisites. Knowledge of linear programming at the level of CO 250 or higher.

Textbook. There is no required textbook.

Course website. math.uwaterloo.ca/~lpoirrier/co370.

We will also use UWaterloo's LEARN website (learn.uwaterloo.ca) as an on-line gradebook so you can keep track of all of your grades in the course.

Lectures. AL 113, Tue-Thu 10:00 – 11:20

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Assignments and exams. There will be 4 assignments. The questions and due date will be communicated at least a week in advance. No late submissions will be accepted, regardless of circumstances. Some of the assignments will involve numerical data. We will use the domain-specific language JuMP to encode our models. JuMP is based on the Julia language, which is popular with data-science applications. No prior knowledge of Julia is expected, but you will be asked to learn the basics for the assignments. An introduction will be covered in class.

There will be one midterm and one final exam. Both are closed-book, no calculators. The first midterm will be held Tuesday, October 29th at 10am in the regular classroom (AL 113). The final exam will be comprehensive and will be held on Thursday, December 19th, 2019, from 7:30pm to 10:00pm (i.e. 19:30 – 22:00), in DC 1351.

Missed exams will count as 0 unless suitable medical documentation is provided. There will not be any make-up exams.

The total grade will be homeworks 20%, midterm 30%, final 50%, or homeworks 20%, midterm 15%, final 65%, whichever is higher.

Course contents.

I. Linear optimization

- Definition and simple models
- Example 1: NH_3 and NH_4Cl .
- Example 2: Blending problem
- Example 3: Multiperiod investment problem
- Max s - t flow problem
 - Refresher: directed graphs
 - Max s - t flow problem statement and formulation
 - Integer solutions property
 - Bipartite matchings
- Min-cost flow problem
 - Min-cost flow problem statement and formulation
 - Application example: transshipment problem
 - Integer solutions property
 - Application example: Bus scheduling

II. Integer optimization

- Example 1: knapsack problem
- Example 2: bin packing problem
- IP modeling tricks (part I)
- Example 3: Sports scheduling problem
- IP modeling tricks (part II)
 - Discrete-valued variables
 - Piecewise-linear functions
 - Union of polyhedra

III. Solving LPs

- A. Standard equality form
- B. Finding a solution to $Ax = b$
- C. Solving $\min\{c^T x : Ax = b, x \geq 0\}$
 - Definitions (basis)

- Theorem: Existence of optimal basic feasible solutions
- Theorem: Basic feasible solutions correspond to vertices
- Naive algorithm for LP
- Observations (O1, O2, O3)
- Simplex tableau
- Pivots
- Simplex method

D. Duality

E. Bases and duality

F. Sensitivity analysis

- Changes to right-hand sides
- Changes to costs
- Changes to coefficients

G. Dual simplex method

IV. Solving IPs

- Branching
- Branch and bound
- Totally unimodular matrices
 - Theorem 1 (Laplace expansion)
 - Theorem 2 (Cramer's rule)
 - Theorem 3 (Square linear systems with unimodular matrices)
 - Theorem 4 (SEF with unimodular $m \times m$ submatrices)
 - Theorem 5 (Operations that preserve total unimodularity)
 - Theorem 6 (LPs in inequality form with a TU matrix)
 - Theorem 7 (Special class of TU matrices: at most one +1/-1 per column)
 - Theorem 8 (Min-cost flow formulation has a TU matrix)
 - Theorem 9 (Max flow formulation has a TU matrix)

V. Other types of problems

- Satisfiability (SAT)
 - Definitions (Boolean variable, Boolean formula, assignment, (un)satisfiable formula, SAT problem)
 - Example (wedding dinner)
 - Boolean reformulations (Boolean identities)
 - Conjunctive normal form (CNF)
 - Backtracking
 - Reformulating a SAT problem as an IP
 - Variants of SAT: constraint programming (CP), satisfiability modulo theories (SMT)

- Nonlinear optimization
 - Black-box / oracle model
 - Definitions (global and local minimizers, gradient)
 - Gradient descent
- Application: Classification / labeling problem
 - Problem statement
 - Neural network (NN) approach
 - Training a neural network
 - Cost function
 - Differentiation
 - Efficient training of a neural network (differentiation for layered NN, stochastic gradient descent, learning rate)

Discipline, appeals, accessibility

Academic Integrity. In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. For more information, check <http://www.uwaterloo.ca/academicintegrity>.

Grievance. A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, <http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm>. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline. A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, <http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm>. For typical penalties check Guidelines for the Assessment of Penalties, <http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm>.

Appeals. A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, <http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm>.

Students with disabilities. The AccessAbility Services, located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with them at the beginning of each academic term.