II. INTEGER (LINEAR) PROGRAMMING

Example : Knapsack problem

An airplane can carry b kilograms of packages

There are N types of packages.

$\forall j = 1, \dots, N$, a package of type j has:

- weight a_j kilograms
- value c_j dollars

What is the maximum value that one airplane can carry?

example

	Weight	Value
crate of bananas	10	40
postal mail	2	7
electronics	17	200
fashion items	7	30
sum	≤ 900	maximize

VAR. x_j : number of packages of type j.

MODEL:

$$\max \sum_{j=1}^{N} c_j x_j$$

$$\text{s.t.} \sum_{j=1}^{N} a_j x_j \leq b$$

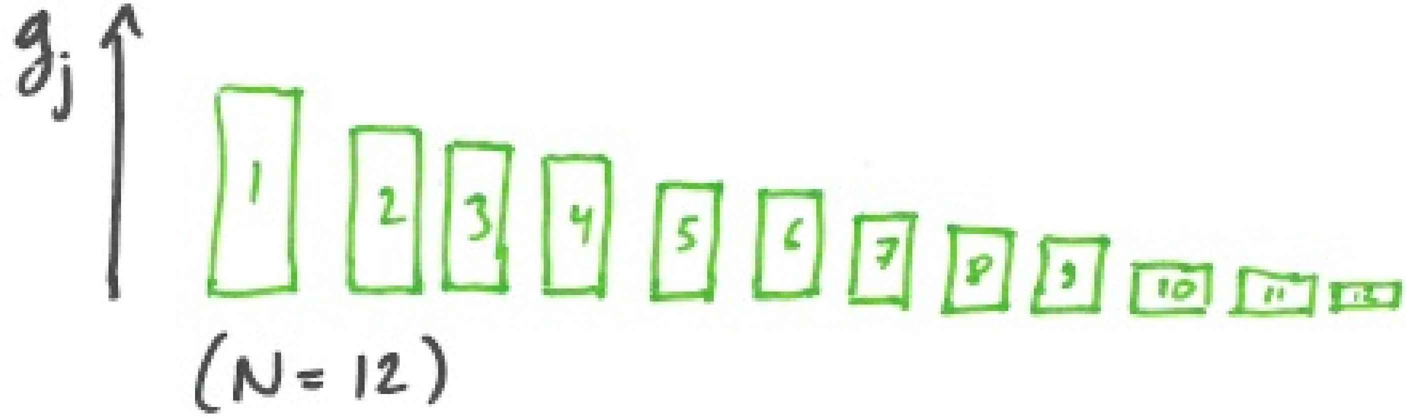
$$x_j \geq 0 \quad \forall j = 1, \dots, N$$

$$x_j \in \mathbb{Z} \quad \forall j = 1, \dots, N$$

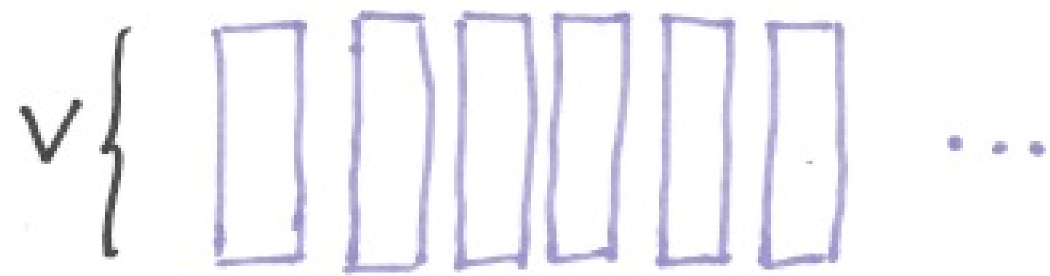
Example: Bin packing

We are given N objects of sizes $g_1, \ldots, g_N$
We have bins of a fixed size V.
Place the objects in as few bins as possible.

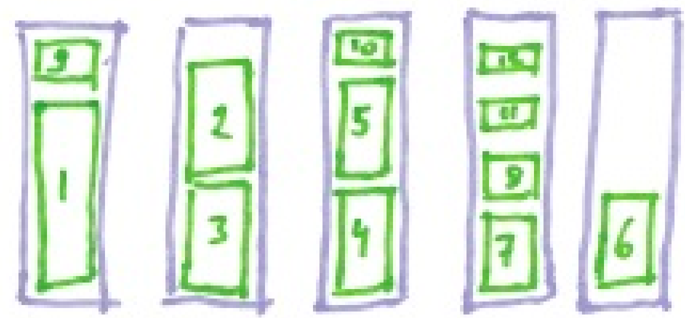
objects :



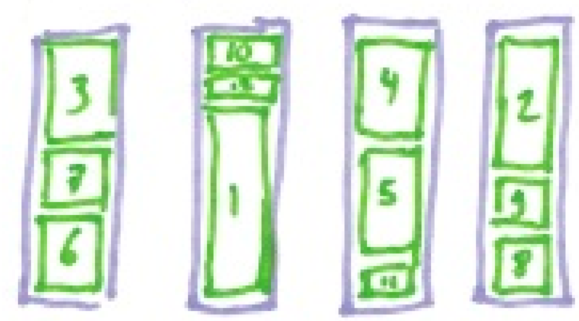
bins :



a feasible solution (5 bins) :



an optimal solution (4 bins) :



Remark

If $g_j > V$ for any $j \in \{1, \dots, N\}$ then problem infeasible.

So we assume $g_j \leq V$, $\forall j$.

$\Rightarrow$ We need at most N bins.

objects

$$
\begin{array}{c|cccccccccccc|c}
x_{ij} & j= 1 & & & & & & & & & & & N & \\
\hline
i=1 & & & & & & & & & & & & & \\
\text{bins} & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & \cdots & & & 1 \\
N & 0 & \cdots & & & & & & & & & & 0 & 0
\end{array}
$$

<u>VAR.</u>

$$x_{ij} = \begin{cases} 1 & \text{if object } j \text{ is placed in bin } i \\ 0 & \text{otherwise,} \end{cases}$$

$$\forall j = 1, \dots, N \ , \quad \forall i = 1, \dots, N$$

$$y_i = \begin{cases} 1 & \text{if bin } i \text{ is used} \\ 0 & \text{if bin } i \text{ is empty} \end{cases} , \quad \forall i = 1, \dots, N$$

<u>MODEL:</u>

$$\min \quad \sum_{i=1}^{N} y_i$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{ij} = 1 \ , \quad \forall j = 1, \dots, N$$

$$\cancel{\sum_{j=1}^{N} g_j \, x_{ij} \leq V, \quad \forall i = 1, \dots, N}$$

$$\sum_{j=1}^{N} g_j \, x_{ij} \leq V \cdot y_i \ , \quad \forall i = 1, \dots, N$$

$$x_{ij} \in \{0, 1\}, \quad \forall i,j \in \{1, \dots, N\}$$
$$y_j \in \{0, 1\}, \quad \forall j \in \{1, \dots, N\}$$