Sports scheduling

$T = 20$ teams

every team plays every other team twice: once "home", once "away"

$M = 38$ match days.

minimize the (largest) number of consecutive away games.

VAR:

$$x_{ijd} = \begin{cases} 1 & \text{if team } i \text{ plays } j \text{ at } i\text{'s home on day } d. \\ 0 & \text{otherwise} \end{cases}$$

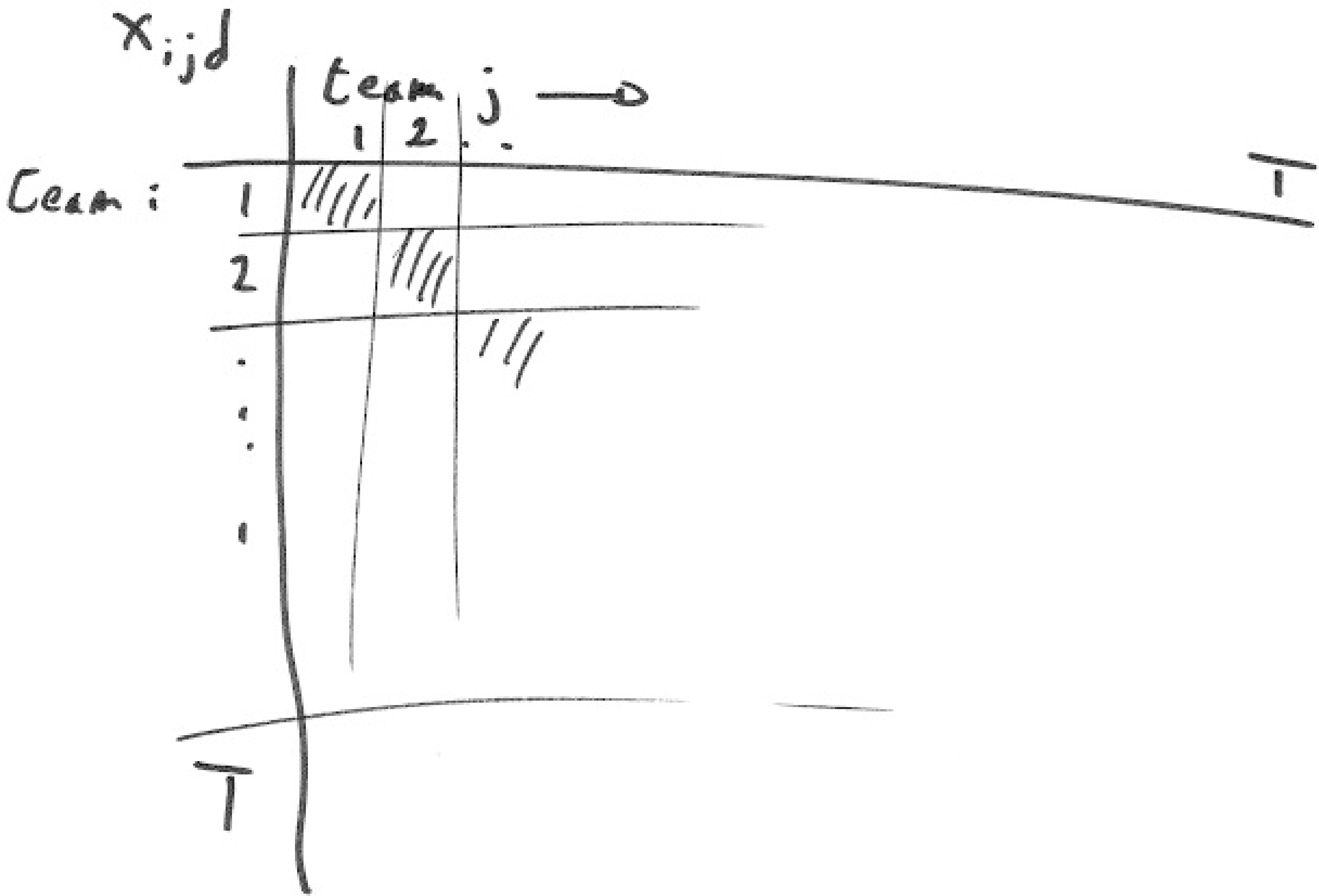
$\forall i, j = 1, \ldots, T, \; \forall d = 1 \ldots P$
$i \neq j$

$$z_{id} = \begin{cases} 1 & \text{if team } i \text{ plays away on day } d \\ 0 & \text{otherwise} \end{cases}$$

$\forall i = 1, \ldots, T, \; \forall d = 1, \ldots, M$

y_{id} = number of consecutive away games for i up to day d.
$\forall i, \forall d$

U = largest y_{id} of any team


x_ijd
team j
1
2
T
team i
1
2
T

example: $(i=1)$

	$d=$ 1	2	3			.	.	.			M
z_{id}	0	0	1	1	0	1	1	1	0	...	0
y_{id}	0	0	1	2	0	1	2	3	0	...	0

y_{id} = number of consecutive away games up to day d

$$y_{id} = \begin{cases} 0 & \text{if } z_{id} = 0 \\ y_{i(d-1)} + 1 & \text{if } z_{id} = 1 \end{cases}$$

$$y_{id} \geq y_{i(d-1)} + 1 - d\,(1 - z_{id})$$

MODEL:

$$\min \quad U$$

$$\text{s.t.} \quad \sum_{d=1}^{M} x_{ijd} = 1 \qquad \forall i, j, i \neq$$

$$\sum_{\substack{j=1 \\ j \neq i}}^{T} x_{ijd} + \sum_{\substack{j=1 \\ j \neq i}}^{T} x_{jid} \leq 1 \qquad \forall i, \forall d$$

$$z_{id} = \sum_{\substack{j=1 \\ j \neq i}}^{T} x_{jid} \qquad \forall i, \forall d$$

$$y_{i1} = z_{i1} \qquad \forall i$$

$$y_{id} \geq y_{i(d-1)} + 1 - d(1 - z_{id}) \qquad \forall i, \ \forall d \geq 2$$

$$U \geq y_{id} \qquad \forall i, \forall d$$

$$y_{id} \geq 0 \qquad \forall i, \ \forall d$$

$$x_{ijd} \in \{0, 1\} \qquad \forall i, \forall j, \forall d$$

Sports scheduling (big-M model)

$$
\begin{aligned}
\min\ & u \\
\text{s.t.}\ & \sum_{d=\{1,\dots,M\}} x_{ijd} = 1 && \forall i \neq j \\
& \sum_{j \in \{1,\dots,T\} \setminus \{i\}} x_{ijd} + \sum_{j \in \{1,\dots,T\} \setminus \{i\}} x_{jid} \leq 1 && \forall i, \forall d \\
& z_{id} = \sum_{j \in \{1,\dots,T\} \setminus \{i\}} x_{jid} && \forall i, \forall d \\
& y_{i1} = z_{i1} && \forall i \\
& y_{id} \geq y_{i(d-1)} + 1 - d\,(1 - z_{id}) && \forall i, \forall d > 1 \\
& u \geq y_{id} && \forall i, \forall d \\
& y_{id} \geq 0 && \forall i, \forall d \\
& x_{ijd} \in \{0, 1\} && \forall i, \forall j, \forall d
\end{aligned}
$$

example ($i=1$)

	$d=$ 1	2	3	$\cdots$						M
z_{id}	0	0	1	1	0	1 1	1	0	$\cdots$	0

$$y_{ide} = \begin{cases} 1 & \text{if } z_{id} = z_{i(d+1)} = \cdots = z_{ie} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$(e-d+1)$ consecutive away games

$\forall i = 1,\dots,T, \quad \forall d,e = 1,\dots,M$
$e > d$

MODEL:

$$y_{ide} \geq \sum_{\delta=d}^{e} z_{i\delta} - (e-d) \qquad \forall i, \forall e > d$$

$$u \geq (e-d+1)\, y_{ide} \qquad \forall i, \forall e > d$$

Sports scheduling (y_{ide} model)

$$
\begin{aligned}
\min\ & u \\
\text{s.t.}\ & \sum_{d=\{1,\dots,M\}} x_{ijd} = 1 && \forall i \neq j \\
& \sum_{j\in\{1,\dots,T\}\setminus\{i\}} x_{ijd} + \sum_{j\in\{1,\dots,T\}\setminus\{i\}} x_{jid} \leq 1 && \forall i, \forall d \\
& z_{id} = \sum_{j\in\{1,\dots,T\}\setminus\{i\}} x_{jid} && \forall i, \forall d \\
& y_{ide} \geq \sum_{\delta\in\{d,\dots,e\}} z_{i\delta} - (e-d) && \forall i, \forall e > d \\
& u \geq (e-d+1)\, y_{ide} && \forall i, \forall e > d \\
& y_{ide} \geq 0 && \forall i, \forall e > d \\
& x_{ijd} \in \{0,1\} && \forall i, \forall j, \forall d
\end{aligned}
$$