Question 1 Consider a company that buys oils and mixes them in order to obtain a final product. The different types of oils that can be bought are shown in the following table, along with their features:

	Name	Hardness
Vegetable Oil	VEG1	8.4
	VEG2	7.1
Non Vegetable Oil	OIL1	2.1
	OIL2	3.8
	OIL3	5.2

Every oil can be bought at the beginning of each season at the following prices ($ per barrel):

	VEG1	VEG2	OIL1	OIL2	OIL 3
Spring	100	110	120	100	105
Summer	120	120	100	80	105
Fall	100	130	120	90	85
Winter	110	100	110	110	115

All the oils have to be refined before use. During any given season, we can refine at most 150 barrels of vegetable oil and at most 180 barrels of non-vegetable oil. The final product will be sold at 160$ per barrel. However, the hardness of the final product has to be between 4.0 and 5.1. (The hardness of the final product is given by the linear combination of the hardness of the oils used to produce it).

(a) Formulate the problem of maximizing the profit of the company for the Spring season (you can ignore the data for the other seasons for this question).

(b) Formulate the problem of maximizing the profit of the company for the entire year, considering that the company can store up to 500 barrels of oil in a storage tank, at a cost of 5$ per barrel in each season, and use these barrels in later seasons. At the beginning of the Spring season the storage tank is empty.

Question 2 A company has 10 workers that it can dispatch to 3 different stores. Once it decides to send a worker to a store, that worker must spend the whole day in that store. Dispatching a worker to the first, second, and third store costs respectively $30, $40, and $50 per worker. Each store i has a number of customers d_i, and every customer served will spend $10. Every worker can serve a maximum of 50 customers per day. Write an integer program that can be used to maximize the profit the company will obtain by serving customers.

Question 3 A company is considering opening factories to produce a single product in a set of cities $F = \{1, \ldots, f\}$. Each factory $i \in F$ has a nominal capacity to produce p_i units of a given product. The fixed cost of keeping each factory open is c_i, for all $i \in F$. The country is divided into a set of regions $R = \{1, \ldots, r\}$ and each region $j \in R$ requires q_j units of the product. The costs of shipping one unit from a factory to a region are π_{ij} for all $i \in F$, $j \in R$. Formulate an IP to minimize the costs.