Question 1 Consider the following LP:

$$\begin{array}{rrcrcrcl}
\min & 4x_1 & - & x_2 & + & 2x_3 & & \\
\text{s.t.} & 2x_1 & + & x_2 & & & \geq & 24 \\
 & & & x_2 & + & 3x_3 & \geq & 12 \\
 & x_1 & & & + & \boldsymbol{t}.x_3 & \geq & 5 \\
 & x_1 & - & x_2 & & & \geq & -12 \\
 & x_1 & , & x_2 & , & x_3 & \geq & 0
\end{array}$$

where $\boldsymbol{t} \in \mathbb{R}$ has some constant value. The value of $\boldsymbol{t}$ was hidden from us, but we know that the problem has a dual optimal solution

$$\left(\frac{1}{3}, \quad 0, \quad 2, \quad \frac{4}{3}\right).$$

1. Find the values of x_1^* and x_2^* for an optimal solution.
2. Find the optimal objective function value.
3. Find the value of x_3^* corresponding to x_1^* and x_2^* found above.
4. Find the value of $\boldsymbol{t}$.

Question 2 A company has to serve 100 customers by satisfying their demand for a given product. For this reason, it is considering opening factories in 40 different locations. Opening factory i, costs f_i dollars. Each customer j has a demand of d_j units of the product. If factory i is open, it can serve customer j at a cost of c_{ij} dollars per unit of product j. Each factory i can produce at most u_i units of the product, and we have the following additional constraints:

i. If factory i is open, then it must produce at least l_i units of the product.

ii. If factories 3 and 4 open, then they must not serve the same client. (ex: if factory 3 serves client 1, then factory 4 cannot serve client 1)

iii. If factories 16 and 18 do not open, then either factory 20 must open or factory 23 must not open.

1. Formulate an IP that the company can use to determine how to satisfy all demands at a minimum cost.
2. Suppose now that the cost of factory i serving x units of the demand of customer j is not linear anymore on x. Instead, it is computed as

$$c(x) = \begin{cases} 10x & , \text{ if } x \in [0, u_i/3] \\ 5x + 5u_i/3 & , \text{ if } x \in [u_i/3, 2u_i/3] \\ 6x + 3u_i/3 & , \text{ if } x \in [2u_i/3, u_i] \end{cases}$$

How would you change your model to take into account these new costs?