Math 115 Spring 2015: Assignment 1

Due: at the tutorial Thursday 5/14

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks. Answers that do not have proper justifications may not receive any credit. Whenever possible, leave your answer in exact form. For example, you should leave $\sqrt{2}$ and $\arccos\left(\frac{\sqrt{10}}{7}\right)$ as is, and not write 1.4142 and 1.1021.

Notice: The solution to question 1c uses material that will be covered in class on Monday 5/11. Answering it correctly will give you bonus marks.

1. Consider the two vectors $\vec{u} = \begin{bmatrix} -2 \\ 4 \\ 4 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 2 \\ -4 \\ 2 \\ 5 \end{bmatrix}$.

   (a) [3 marks] Determine a vector of length 3 that has the same direction as $\vec{u}$.

   (b) [3 marks] Determine the angle between $\vec{u}$ and $\vec{v}$.

(c) [4 bonus marks] Write $\vec{v}$ as the sum of two nonzero orthogonal vectors, one of which is a scalar multiple of $\vec{u}$.

2. [3 marks] Let $\vec{u} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$. Determine the set of all vectors $\vec{x}$ such that $||\vec{u} - \vec{x}|| = ||\vec{v} - \vec{x}||$ (i.e. the distance between $\vec{x}$ and $\vec{u}$ is the same as the distance between $\vec{x}$ and $\vec{v}$).

3. For each of the following statements, either prove that it is true, or find a counterexample to prove that it is false.

   Proposition 1. *[3 marks] Let $\vec{u}, \vec{v} \in \mathbb{R}^n$. Then, $||\vec{u} + \vec{v}||^2 = ||\vec{u}||^2 + ||\vec{v}||^2$.*

   Proposition 2. *[4 marks] Let $\vec{u}, \vec{v} \in \mathbb{R}^n$. Then, $||\vec{u} + \vec{v}||^2 + ||\vec{u} - \vec{v}||^2 = 2||\vec{u}||^2 + 2||\vec{v}||^2$.*