Math 115 Spring 2015: Assignment 2

Due: at the tutorial Thursday 5/21

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks. Answers that do not have proper justifications may not receive any credit. Whenever possible, leave your answer in exact form. For example, you should leave $\sqrt{2}$ and $\arccos\left(\frac{\sqrt{10}}{7}\right)$ as is, and not write 1.4142 and 1.1021.

1. Determine whether or not the following sets are subspaces in their respective vector spaces. If so, prove it using the definition of subspaces. If not, provide a counterexample where a rule of subspaces is violated.

   (a) [2 marks] $S = \{\vec{x} \in \mathbb{R}^2 \mid x_1 + 2x_2 = 0 \text{ and } x_1 - 3x_2 = 1\}$.

   (b) [2 marks] $T = \{\vec{x} \in \mathbb{R}^3 \mid \vec{x} \cdot \vec{v} \geq 0\}$, where $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

2. Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} -3 \\ -2 \\ -3 \end{bmatrix}$, and $\vec{x} = \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$.

(a) [3 marks] Show that $\{\vec{u}, \vec{v}, \vec{w}, \vec{x}\}$ is linearly dependent.

(b) [4 marks] Find a basis for $\text{span}\{\vec{u}, \vec{v}, \vec{w}, \vec{x}\}$, i.e. a set of vectors $\{\vec{v}_1, \ldots, \vec{v}_k\}$ such that (a) $\{\vec{v}_1, \ldots, \vec{v}_k\}$ is linearly independent, and (b) $\text{span}\{\vec{v}_1, \ldots, \vec{v}_k\} = \text{span}\{\vec{u}, \vec{v}, \vec{w}, \vec{x}\}$. (Note: k will be smaller than 4, so it could be 1, 2 or 3.)

3. For each of the following statements, either prove that it is true, or find a counterexample to prove that it is false.

   (a) [3 marks] Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$. If $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly dependent, then $\vec{u} \in \text{span}\{\vec{v}, \vec{w}\}$.

   (b) [3 marks] Let $\vec{u}, \vec{v} \in \mathbb{R}^n$ be two nonzero vectors (i.e. $\vec{u} \neq \vec{0}$ and $\vec{v} \neq \vec{0}$). If $\vec{u}$ and $\vec{v}$ are orthogonal, then $\{\vec{u}, \vec{v}\}$ is linearly independent.

   (c) [3 marks] Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ be three nonzero vectors. If (a) $\vec{u}$ is orthogonal to $\vec{v}$, and (b) $\vec{u}$ is orthogonal to $\vec{w}$, then $\{\vec{u}, \vec{v}, \vec{w}\}$ is linearly independent.