Math 115 Spring 2015: Assignment 4

Due: at the tutorial Thursday 6/4

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks.

1. [5 marks] Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & 1 \end{bmatrix}$. Compute AA^T and A^TA.

2. [5 marks] Given scalars $t_1, t_2 \in \mathbb{R}$, find a matrix $A \in \mathbb{R}^{3\times 3}$ such that $A \cdot \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} (x_1 + t_1) \\ (x_2 + t_2) \\ 1 \end{bmatrix}$, for all $x_1, x_2 \in \mathbb{R}$. **Note:** The matrix must be the same for all values of x_1 and x_2, but the scalars t_1 and t_2 are constant, so they may appear in A. **Hint:** Expand the product $A\vec{x}$ in function of the scalar elements of A and $\vec{x}$.

3. [5 marks] Given a vector $\vec{v} \in \mathbb{R}^2$, find a matrix $A \in \mathbb{R}^{2\times 2}$ such that $A \cdot \vec{x} = \text{proj}_{\vec{v}}\,\vec{x}$, for all $x \in \mathbb{R}^2$. **Note:** The matrix must be the same for all $\vec{x}$, but $\vec{v}$ is constant, so v_1 and v_2 may appear in A. **Hint:** Expand $\text{proj}_{\vec{v}}\,\vec{x}$ and the product $A\vec{x}$.

4. [5 marks] A matrix B is *symmetric* if $(B)_{ij} = (B)_{ji}$ for all i and j. Show that, for any matrix $A \in \mathbb{R}^{m\times n}$, the product (A^TA) is (a) defined, (b) a square matrix, and (c) a symmetric matrix.