Math 115 Spring 2015: Assignment 5

Due: at the tutorial Thursday 6/11

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks.

1. (a) [2 marks] Find a matrix $A \in \mathbb{R}^{2\times 2}$ such that $\vec{y} = A\vec{x}$, where $\vec{y}$ is $\vec{x}$ rotated by an angle of $\frac{2}{3}\pi$ (counterclockwise around the origin), for any $\vec{x} \in \mathbb{R}^2$.

   (b) [2 marks] Find a matrix $B \in \mathbb{R}^{2\times 2}$ such that $\vec{z} = B\vec{y}$, where z_1 is y_1 scaled by a factor 3 and z_2 is y_2 scaled by a factor 2, for any $\vec{y} \in \mathbb{R}^2$.

   (c) [2 marks] Find a matrix $C \in \mathbb{R}^{2\times 2}$ such that $\vec{w} = C\vec{z}$, where $\vec{w}$ is $\vec{z}$ rotated by an angle of $\frac{-2}{3}\pi$ (counterclockwise around the origin, i.e. $\frac{2}{3}\pi$ clockwise), for any $\vec{z} \in \mathbb{R}^2$.

   (d) [2 marks] Find a matrix $G \in \mathbb{R}^{2\times 2}$ such that $\vec{w} = G\vec{x}$, where $\vec{w}$ is $\vec{x}$ that is rotated by $\frac{2}{3}\pi$ and then scaled with factors 3 and 2, and then rotated by $\frac{-2}{3}\pi$. Note that this amounts to performing on $\vec{x}$ all three transformations found in points (a), (b) and (c), successively.

2. [3 marks] Find a matrix $A \in \mathbb{R}^{2\times 2}$ that has no zero elements, such that $\vec{x} = A^k\vec{x}$. Note that $A^k = A \cdot A \cdots A$ where there are k factors A. For example $A^3 = A \cdot A \cdot A$. **Hint:** Think about a geometric transformation that, when applied k times on the vector $\vec{x}$, gives back the vector $\vec{x}$ itself. k may appear in some form in the matrix.

3. The matrix $A \in \mathbb{R}^{4\times 7}$ and its reduced row echelon form B are given as follows:

$$A = \begin{bmatrix} 1 & -3 & 0 & 1 & 4 & 1 & -5 \\ 0 & 0 & -1 & 5 & -9 & -1 & 4 \\ 3 & -9 & -1 & 8 & 3 & 1 & -5 \\ -1 & 3 & 1 & -6 & 5 & -1 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -3 & 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 1 & -5 & 9 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

   (a) [2 marks] Determine a basis for the columnspace of A.

   (b) [2 marks] Determine a basis for the rowspace of A.

   (c) [5 marks] The set $S = \{x \in \mathbb{R}^7 | A\vec{x} = \vec{0}\}$ is the set of all solutions to the system $A\vec{x} = \vec{0}$. This set S is a subspace. Determine a basis for S. **Hint:** Find the general solution to $A\vec{x} = \vec{0}$, and write it as a vector equation.