Math 115 Spring 2015: Assignment 6

Due: at the tutorial Thursday 6/25

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks.

1. [5 marks] Let $G = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 1 \\ 5 & 3 & 0 \end{bmatrix}$. Compute G^{-1}.

2. [4 marks] Let
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 2 & 0 & 1 \\ 2 & 3 & 5 & 1 & 2 \\ 4 & 1 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 & 0 \end{bmatrix}.$$
Compute $\det(A)$. **Hint:** choose carefully the columns or rows to expand in order to reduce your work.

3. [5 marks] Let
$$B = \begin{bmatrix} 1 & 4 & 5 & 3 \\ 0 & 2 & 3 & 3 \\ 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$
Notice that B is upper-triangular (i.e. all elements below the diagonal are zero). Use the cofactor expansion of determinants to show (on this example) that the $\det(B)$ is simply the product of the diagonal elements of B.

4. For each of the following statements, either prove that it is true, or find a counterexample to prove that it is false.

   (a) [3 marks] If A and B are $n \times n$ invertible matrices, then $A + B$ is also invertible.

   (b) [3 marks] If A and B are $n \times n$ invertible matrices and $(AB)^2 = A^2B^2$, then $AB = BA$.