Math 115 Spring 2015: Assignment 7

Due: at the tutorial Thursday 7/2

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks.

1. [5 marks] Let

$$A = \begin{bmatrix} 4 & 2a+2b & 2a \\ 2 & a+b+1 & a+b \\ 2 & a+b+1 & a+b+1 \end{bmatrix},$$

where $a, b \in \mathbb{R}$. The determinant of A is a constant independent of a and b. Find its value. **Hint:** Compute the REF of A.

2. [5 marks] Let

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ t \cdot b_{31} & t \cdot b_{32} & t \cdot b_{33} \end{bmatrix}$$

for $t \in \mathbb{R}$. Use Cramer's rule to prove that

$$\text{if} \quad B^{-1} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}, \quad \text{then} \quad C^{-1} = \begin{bmatrix} g_{11} & g_{12} & \frac{1}{t} \cdot g_{13} \\ g_{21} & g_{22} & \frac{1}{t} \cdot g_{23} \\ g_{31} & g_{32} & \frac{1}{t} \cdot g_{33} \end{bmatrix}.$$

3. [5 marks] For each of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$, determine whether or not it is an eigenvector of E. If so, determine its corresponding eigenvalue.

$$E = \begin{bmatrix} 12 & -18 & 6 \\ 13 & -17 & 6 \\ 11 & -9 & 4 \end{bmatrix}, \quad \vec{v}_1 = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}.$$

4. [5 marks] Prove that if λ is an eigenvalue of $D \in \mathbb{R}^{n \times n}$, then λ^2 is an eigenvalue of $D \cdot D$.