Math 115 Spring 2015: Assignment 8

Due: at the tutorial Thursday 7/9

Last name:

First name:

ID number:

Note: You need to show all the steps and the reasoning in obtaining your answers in order to receive full marks.

1. Let
$$A = \begin{bmatrix} 1 & 6 & -3 \\ 0 & 4 & 0 \\ -3 & 6 & 1 \end{bmatrix}.$$
   (a) [5 marks] Find the eigenvalues of A. For each eigenvalue, give the algebraic multiplicity, determine a basis of the corresponding eigenspace, and give the geometric multiplicity.

   (b) [1 mark] Diagonalize A by finding an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$

2. [5 marks] Determine a matrix $B \in \mathbb{R}^{2\times 2}$ for which $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$ are eigenvectors, with corresponding eigenvalue -4 and 2, respectively.

3. [4 marks] Let $C \in \mathbb{R}^{n\times n}$ be a diagonalizable matrix that has n distinct eigenvalues. Prove that $\det(C)$ is the product of the n eigenvalues of C.

4. [5 marks] Let $G \in \mathbb{R}^{n\times n}$ be a diagonalizable matrix such that $Q^{-1}GQ = \text{diag}(\mu_1, \ldots, \mu_n)$. Find an expression of G^4 that does not involve G (it may involve, Q and $\mu_1, \ldots, \mu_n$). Give the eigenvalues of G^4.