

# Linear algebra properties and identities

Here is a list of useful identities and properties for computing with vectors and matrices. This list is *not* exhaustive. You are not allowed to read course material (including this list) at the exam.

## 1 Vectors

Let  $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$  and  $s, t \in \mathbb{R}$ .

- $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
- $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$
- $s(t\vec{a}) = (st)\vec{a}$
- $(s + t)\vec{a} = s\vec{a} + t\vec{a}$
- $s(\vec{a} + \vec{b}) = s\vec{a} + s\vec{b}$
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $(s\vec{a}) \cdot \vec{b} = s(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (s\vec{b})$
- $(s\vec{a}) \cdot (t\vec{b}) = st(\vec{a} \cdot \vec{b})$

## 2 Matrices

Let  $A, B, C \in \mathbb{R}^{m \times n}$ ,  $G \in \mathbb{R}^{p \times q}$ ,  $H \in \mathbb{R}^{q \times m}$ ,  $\vec{v} \in \mathbb{R}^n$  and  $s, t \in \mathbb{R}$ .

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $s(tA) = (st)A$
- $s(A + B) = sA + sB$
- $(s + t)A = sA + tA$
- $H(sA) = s(HA)$
- $A(s\vec{v}) = s(A\vec{v})$
- $H(A\vec{v}) = (HA)\vec{v}$
- $G(HA) = (GH)A$
- $(sH) \cdot (tA) = stHA$
- $(sA)^T = s(A^T)$
- $(A + B)^T = A^T + B^T$
- $(HA)^T = A^T H^T$