Math 115 Spring 2015: Quiz 6

Solutions

1. Compute the inverse of

$$B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Solution: The RREF of

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right].$$

is

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 1 & -2 & 0 \end{array}\right].$$

so

$$B^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & 1 \\ 1 & -2 & 0 \end{bmatrix}.$$

2. Let

$$C = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

The value of $\det(C)$ is independent of θ. Find that value. **Hint:** For any angle $\alpha \in \mathbb{R}$, $\cos^2\alpha = 1 - \sin^2\alpha$.

Solution:

$$\det\left(\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\right) = \cos\theta\cos\theta + \sin\theta\sin\theta = \cos^2\theta + \sin^2\theta = 1$$

3. Let

$$D = \begin{bmatrix} 1 & 0 & 2 \\ a & b & 3 \\ 2 & 0 & 1 \end{bmatrix}.$$

Determine the values of $a, b \in \mathbb{R}$ such that $\det(D) = 0$.
Solution: We expand along the third column to obtain

$$\det(D) = b.\det\left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\right) = b.(1.1 - 2.2) = -3b$$

so $\det(D) = 0$ if and only if $b = 0$ (regardless of the value of a).

4. Assume that $A \in \mathbb{R}^{n\times n}$ is such that $3(A \cdot A \cdot A) + 2(A \cdot A) = I$. Prove that A^{-1} exists. **Hint:** Find an expression such that A multiplied by this expression is I. This proves that the expression is A^{-1}.

Solution: We rewrite $3(A \cdot A \cdot A) + 2(A \cdot A) = I$ as $A \cdot 3A \cdot A + A \cdot 2A = I$ and factor out A to obtain

$$(A).(3A \cdot A + 2A) = I$$

Therefore, if the expression $(3A \cdot A + 2A)$ is defined, then it is the inverse of A. Since A is square, the expression is well-defined, so the inverse of A exists.